Inflation targeting, the zero lower bound and post-crisis monetary policy

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June 2012

Abstract

This paper addresses recent developments in monetary policy theory in the context of a binding Zero Lower Bound and discusses the possible evolution of monetary policy after the Great Recession. We start from Olivier Blanchard's suggestion that a higher inflation target and correspondingly higher interest rates would offer larger wiggle room for Central Banks to stimulate the economy through monetary easing without hitting the ZLB and might thus prove to be a desirable policy. Using a New-Keynesian DSGE framework and including positive steady state inflation, we investigate if having a higher permanent inflation target would improve welfare and find that this is unlikely. Furthermore, we address the possibility of having temporary higher inflation targets and the effect this could have on economic fundamentals. Finally, we discuss whether simple inflation targeting suffices or if monetary policy might evolve in the aftermath of the crisis towards including several objectives and/or instruments, so as to better respond to future economic downturns.

*The author would like to thank Professor Patrick Artus for his invaluable help and Professors Florin Bilbiie, Luca Benati, Jean-Olivier Hairault and Marco Del Negro for useful comments and suggestions

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1 Introduction

The field of macroeconomics has faced a significant challenge brought about by what is now known as the Great Recession or the Lesser Depression which started in 2008. By all accounts, it has failed both in predicting this crisis and, more importantly, in offering a clear and unified recipe for overcoming it. The reasons for this stark reality are rather complex and have been detailed by several authors. [1] A general conclusion is that the profession entered a so-called Dark Age, namely it chose to ignore very important developments and hard-won lessons that previous generations of macroeconomists had contributed.

Origins of this contradictory reality can be traced back to the diverging views on the science that emerged in the 1970's when, dissatisfied with the lack of rigor of Keynesian and Hicksian IS-LM models, economists started developing a different modeling approach based on micro-foundations, following the Lucas critique. What the science undoubtedly gained in precision and mathematical rigor, it lost in empirical applicability which was significantly hindered by the unrealistic assumptions which were often the bedrock of the new models. This combined with the fact that economists were no longer well-versed in old-fashion Keynesian analysis which, for better or worse, was empirically accurate and offered clear answers for economic policy in a downturn, generated the unsatisfactory reaction of the profession to the recent developments.

In this context, one of the economic phenomena that had been overlooked but suddenly came back to the forefront with increased relevance was the Zero Lower Bound (henceforth ZLB), which occurs when central banks take their benchmark interest rate as low as possible, in the immediate vicinity of 0. This milestone is important as it signifies the end of the central bank's ability to stimulate the economy through conventional monetary policy - very straightforwardly, cutting interest rates in a depression so as to increase liquidity in the markets. If the economy is still depressed when interest rates reach 0, unconventional monetary policy or fiscal policy are the only two actions which can provide further stimulus.

The experience of Japan, which has been at the ZLB since the beginning of 1995 - so for 17 years- without having the ability to overcome it, has been publicized and studied rather extensively, mostly by Japanese economists. Even so, most macroeconomists still believed in 2008 that the ZLB is a rather exotic occurrence and more of a theoretical curiosity rather than a real threat to any advanced economy. An often-mentioned argument was that, as per

[1] See for example Krugman (2009), Stiglitz (2010), Roubini and Mihm (2011)

Fisher's equation $r = i - \pi^e$, one can lower real (r) rather than nominal (i) interest rates under the 0 threshold by increasing expectations of inflation (π^e), which would be easy enough to do. However, it turned out that central banks can't generate inflation that easily if they are too credible.

While the US has been at the ZLB since 2008, the consequences for policymaking have been significant, because economists and economic decision-makers failed to grasp in due time that the ZLB turns the rules of policy upside down (e.g. deficit-financed stimulus spending does not increase interest rates on sovereign debt in such circumstances). This has lead to a general unwillingness to embark upon government programs that, while risky in normal times, would have proven the good antidote for the crisis. Adequate fiscal stimulus to prop up the gap in demand or unconventional monetary policy were very hard to come by and when countries finally decided to implement them, it was only very reluctantly, when it became clear that conventional alternatives had failed.

Taking all of this into account, some economists have argued that it could be important in the future for central banks to have more wiggle room for using conventional monetary policy, since alternative courses of action are not politically palatable or have not yet generated consensus. After all, nominal interest rates have hit the ZLB so quickly also because, for various reasons, they were not very large in the beginning of the crisis. Thus, in a 2010 International Monetary Fund Staff Note, we find the following quote: "The crisis has shown that large adverse shocks can and do happen. ... Should policymakers therefore aim for a higher target inflation rate in normal times, in order to increase the room for monetary policy to react to such shocks? To be concrete, are the net costs of inflation much higher at, say, 4% than at 2%, the current target range? Is it more difficult to anchor expectations at 4% than at 2%?". [2]

This is the proposal that motivates this paper, as it has others before. In the first part, we set out to investigate if a permanently higher inflation target might indeed prove justified, taking into account the costs and benefits of such a move. To do so, we make use of a New Keynesian DSGE framework, where inflation plays a central part. Building counterfactuals for monetary policy during the recent crisis, we show the extent to which the zero lower bound was binding and assess the possibility of permanently higher steady-state inflation. In the second part, we bring into focus the possibility of aiming for *temporary* higher inflation, while also discussing post-crisis monetary policy.

[2] Blanchard, Dell'Ariccia, Mauro (2010)

2 Literature Review: Inflation and the Zero Lower Bound in New Keynesian Dynamic Stochastic General Equilibrium Models

New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models are modern monetary models of the business cycle, developed after incorporating price stickiness into micro-founded Real Business Cycles models as first developed by Kydland and Prescott (1982). They are now the standard tool in macroeconomic analysis, as they provide an unmatched degree of rigor and a solid framework for welfare analysis, policy evaluation and economic forecasting.

Standard DSGE models are based on the seminal work by Clarida, Gali, Gertler (1999) and Woodford (2003), while Eggertson, Woodford (2003) have provided a classical reference on how to model the ZLB into the DSGE framework. They have shown that optimal monetary policy involves credible commitment by central banks to a desirable course of action, namely basing interest rate policy on a history-dependent price-level targeting rule which would reduce the output loss from a temporary ZLB constraint.

Adam, Billi (2006) have determined optimal monetary policy under commitment while taking into account the ZLB in computing the policy regime. Using historical shock processes for the US economy, the authors find that the ZLB doesn't impose large constraints on optimal monetary policy. They surmise that the ZLB binds infrequently and that positive inflation is not optimal even in this event as there are no welfare losses associated with it. This became the most encountered policy recommendation, as several papers would find similar results even when using slightly differently articulated DSGE frameworks. Furthermore, Adam, Billi (2006) find that, when faced with adverse shocks, central banks must lower nominal interest rates more aggressively due to the presence of the ZLB.

While previous studies had started with an arbitrarily imposed inflation objective and then solved for its effects, Billi (2009) estimates a New Keynesian model log-linearized around zero steady state inflation, with occasionally binding ZLB on nominal interest rates, adding model uncertainty. Policymakers thus aim to maximize the economic welfare through their choice of inflation rate. His results point to optimal inflation rates between 0.7% and 1.4% per year, depending on the degree of uncertainty allowed, which are generally lower values than those advanced by previous research.

One very notable and recent contribution to the topic of optimality of the inflation rate was put forward by Schmitt-Grohé, Uribe (2010), who simulated a richly calibrated model under several assumptions in a model-economy with numerous frictions. They conclude that the ZLB is not an impediment for setting inflation targets near or below zero, the point being that this constraint binds so rarely, that the optimal inflation target in these situations is not very different than the optimal value in the unconstrained model (-0,4% per year). This reinforces the result of Adam, Billi (2006). As pointed out by McCallum (2011), the problem with this approach is that the calibration used implies a zero probability of hitting the ZLB, so that the optimal rate of inflation is unaffected. This is due to the fact that the lower interbank rate is the most relevant for the ZLB problem, whereas Schmitt-Grohé, Uribe (2010) focus only on the risk-free rate.

The assumption that the non-negativity constraint binds infrequently and that its effects dissipate quickly was a leap of faith. The current recession proved that the ZLB can bind and that it can do so for many periods. Ireland (2011) proves this point in a convincing fashion. Comparing the last three recessions in the United States, he concludes that all of them were produced by a combination of technology and preference shocks, which were usually offset by accommodative conventional monetary policy. In 07-09 however, these shocks were unusually persistent given the full extent of the crisis, and thus pushed the nominal interest rates against the ZLB, causing a much deeper output drop that had been necessary. Using estimated counterfactuals for the path of interest rates and output, this paper submits that only a 1% additional cushion in interest rates would have provided the needed monetary stimulus for a solid recovery. A similar conclusion is reached by Williams (2009) who uses counterfactuals for interest rate and unemployment to show that theoretical additional cuts in interest rate after hitting the ZLB would have ensured a much more rapid return to acceptable levels of unemployment in the aftermath of the recession.

An important caveat to these models was that monetary policy was considered independently, without consideration for the possible interplay between fiscal and monetary policies when nominal interest rates are close to zero. Werning (2011) addressed this issue and used a continuous-time version of the standard New Keynesian DSGE model to draw policy rules in such circumstances. Due to the forward-looking nature of inflation, he finds that commitment to higher future inflation when at the ZLB is crucial and that the policy of higher inflation as a precaution for future liquidity traps may be far from optimal, especially if the ZLB binds for a small duration. Thus, Werning (2011) provides a formal explanation for the proposition that future monetary easing is beneficial at the ZLB, which was first offered by Krugman (1998). Furthermore, the paper shows that the optimal mix of fiscal and monetary policies must combine commitment to zero interest rates for an extended period of time and modest transitory fiscal stimulus in the beginning of the ZLB event. When there is a lack of commitment by the central bank, Werning (2011) shows that fiscal intervention plays a much more important part.

There was still an important limit to the use of New Keynesian DSGE models for optimal monetary policy analysis, especially for the issue of inflation targeting at the ZLB. Blanchard's proposition of higher inflation every period as a hedge against the liquidity trap and its effects means positive steady-state inflation in the DSGE framework. However, almost all the models previously used relied on log-linearization around a zero-inflation steady-state. Motivated by this caveat, Coibion, Gorodnichenko, Wieland (2012) solve a New Keynesian DSGE model with positive steady state inflation, deriving implications for welfare analysis. "Using a welfare criterion derived explicitly from the micro-foundations of the model", they study the optimal inflation rate comparing costs and benefits generated by targeting a higher rate of inflation. As opposed to previous models, they assume that higher trend inflation has not only costs (greater price dispersion, increased price volatility, more forward-looking behavior) but also benefits (reduced frequency and cost of hitting the ZLB). Even in this framework, the authors conclude that the optimal inflation rate can't surpass 2% and that higher inflation targets are likely too blunt an instrument to address the costs of the ZLB in an efficient way. Non-standard monetary policy is considered by the authors to be the best way of addressing these costs.

DSGE models continue to be the best tool for evaluating monetary policy, but some qualifications can be added to this statement. It is known that any DSGE model's performance is only as good as the model's assumptions, so precautions must be taken when building such models to avoid assuming away some characteristics of the true economy. As far as the ZLB is concerned, it seems key to us that its existence and possible persistence have to be modeled explicitly. Furthermore, model misspecification through omitted variables may compromise the forecasts generated. In the context of the present analysis, the models provide no role for the existence of credit frictions, banks or debt, which is unlikely to be anodyne. Furthermore, such models rely on the short-term interest rate set through a variation of the Taylor rule as the only transmission channel for monetary policy. No role is given to bank reserves or currency which are often used as alternative transmission channels. Thus, an extended DSGE model including these features, may prove to be even more reliable than the ones currently in use both for business cycle analysis and forecast.

3 The Models

In this section, we will detail two types of approaches to evaluating the optimal inflation rate in New Keynesian DSGE models. Starting from a well-known framework introduced by Clarida, Gali, Gertler (1999) and Woodford (2003), the first uses a log-linearization around a steady state where inflation is presumed to be zero in order to derive Ramsey-optimal monetary policy. Then, following Ireland(2011) and Coibion, Gorodnichenko, Wieland (2012), we introduce positive steady-state inflation to the model and discuss the implications of this.

3.1 Ramsey-Optimal Monetary Policy at the Zero Lower Bound in a New Keynesian DSGE Model

New Keynesian DSGE Models differ from their precursors, RBC Models, by implicitly including price stickiness. But there are several ways in which to formulate the idea that prices may be sticky. Taylor (1979) offered a popular way to model this through staggered contracting. The assumption is that firms set an optimal price contingent on knowing that the price will then be fixed for a number of periods. Calvo (1983) supposed that, in every period, not all firms change their prices, but only a fraction of them, while the others keep them unchanged; the ones that do change must take into account that the price may be fixed for several periods after that.

In this subsection, I use the method introduced by Rotemberg (1982), the logic of which is that firms have to pay a certain quadratic adjustment cost for changing their prices, which is the same for every firm ω. The cost of inflation volatility is measured around a steady-state with zero inflation and is given by:

$$Adj_t(\omega) = \frac{\kappa}{2}\left(\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1\right)^2 C_t$$

where p_{t-1} is given by nature and κ is a positive parameter. The production function of this firm will exhibit constant returns to scale, taking the form:

$$y_t = Z_t L_t$$

Demand from consumers for the output of firm ω is:

$$y_t^D(\omega) = \left(\frac{p_t(\omega)}{P_t}\right)^{-\theta} C_t$$

where θ is the elasticity of substitution. The advantage of using Rotemberg pricing is that the equilibrium will be symmetric, since all firms pay the same cost. Thus relative prices will be unity so firms will produce the same output with the same amount of labor. Profit for a generic firm can then be written:

$$\Psi_t = (1+\tau)y_t^D - w_t L_t - \frac{\kappa}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 C_t$$

The firm will maximize this profit at time t, choosing L_t (labor remunerated at w_t) and p_t (price), subject to $y_t = y_t^D$. τ is a sales subsidy which can be used to correct any steady-state distortions. Solving this problem by taking into account the symmetry of the equilibrium and denoting $1+\pi_t = \frac{P_t}{P_{t-1}}$, we obtain the first order conditions:

$$L_t : \lambda_t = \frac{w_t}{Z_t}$$

$$p_t : (1+\tau)(1-\theta) + \theta\lambda_t - \kappa\pi_t(1+\pi_t) + \beta\kappa E_t[\pi_{t+1}(1+\pi_{t+1})] = 0$$

Z_t is production per unit of labor (the technology), so λ_t, the Lagrange multiplier on the constraint, is the marginal cost while β is the coefficient of inter-temporal preference. From the labor supply decision of households we know that:

$$w_t = C_t v_L(L_t)$$

We can derive this model's New Keynesian Phillips curve, after log-linearizing around a steady state with zero inflation:

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\theta - 1}{\kappa}\mu_t$$

Where $\mu_t = -\lambda_t$ is the mark-up. Consequently we can write the Ramsey problem which is solved to get the optimal inflation rate, assuming a separable and logarithmic in consumption form for the utility function. $v(L)$ is isoelastic, namely: $\frac{v_{LL}(L)L}{v_L(L)} = \varphi$:

$$\max_{C_t,L_t,\pi_t} E_0 \sum_{t=0}^{\infty} \beta^t [lnC_t - v(L_t)] \tag{1.1}$$

s.t.

$$C_t \left(1 + \frac{\kappa}{2}\pi_t^2\right) = Z_t L_t \tag{1.2}$$

$$(1+\tau)(1-\theta) + \theta \frac{C_t v_L(L_t)}{Z_t} - \kappa\pi_t(1+\pi_t) + \beta\kappa E_t[\pi_{t+1}(1+\pi_{t+1})] = 0 \tag{1.3}$$

$$1 = \beta E_t \left[\frac{(1+I_t)C_t}{(1+\pi_{t+1})C_{t+1}}\right] \tag{1.4}$$

$$I_t \geqslant 0 \tag{1.5}$$

where (1.2) is the resource constraint which states that output will be used for consumption and for paying the quadratic adjustment cost, (1.3) is the first-order condition previously derived, (1.4) is the Euler equation, and (1.5) is the non-negativity constraint on the nominal interest rate I_t, which is now included into the model and treated explicitly. The Lagrangian is:

$$\begin{aligned}
&\max_{C_t,L_t,\pi_t} E_0 \sum_{t=0}^{\infty} \beta^t \{lnC_t - v(L_t) + \\
&\quad + \eta_{1,t}\left[Z_t L_t - C_t\left(1 + \frac{\kappa}{2}\pi_t^2\right)\right] + \\
&\quad + \eta_{2,t}\left[(1+\tau)(1-\theta) + \theta\frac{C_t v_L(L_t)}{Z_t} - \kappa\pi_t(1+\pi_t) + \beta\kappa E_t[\pi_{t+1}(1+\pi_{t+1})]\right] + \\
&\quad + \eta_{3,t}\left[1 - \beta E_t\left(\frac{1+I_t}{1+\pi_{t+1}}\frac{C_t}{C_{t+1}}\right)\right] + \\
&\quad + \eta_{4,t} I_t\}
\end{aligned} \tag{1.6}$$

Where $\eta_{1,t}, \eta_{2,t}, \eta_{3,t}, \eta_{4,t}$ are the Lagrange multipliers corresponding to the four constraints. In standard DSGE models, the non-negativity constraint is ignored, thus assuming negative interest rates are possible. Solving a model which includes at least one non-negativity constraint, here the ZLB, requires the use of a different technique than usual, namely the Kuhn-Tucker formulation, explained in Simon, Blume (1994). The optimality conditions are now:

$$\begin{aligned} C_t : & \frac{1}{C_t} - \eta_{1,t}\left(1 + \frac{\kappa}{2}\pi_t^2\right) + \eta_{2,t}\theta\frac{v_L(L_t)}{Z_t} - \\ & - \eta_{3,t}\beta\frac{1+I_t}{1+\pi_t+1}\frac{1}{C_{t+1}} + \eta_{3,t-1}\frac{1+I_{t-1}}{1+\pi_t}\frac{C_{t-1}}{C_t^2} = 0 \end{aligned} \tag{1.7}$$

$$L_t : -v_L(L_t) + \eta_{1,t}Z_t + \eta_{2,t}\theta\frac{C_t v_{LL}(L_t)}{Z_t} = 0 \tag{1.8}$$

$$\begin{aligned} \pi_t : & - \kappa\eta_{1,t}C_t\pi_t - \kappa\eta_{2,t}(1+2\pi_t) + \kappa\eta_{2,t-1}(1+2\pi_t) + \\ & + \eta_{3,t-1}\frac{1+I_{t-1}}{(1+\pi_t)^2}\frac{C_{t-1}}{C_t} = 0 \end{aligned} \tag{1.9}$$

$$I_t : -\eta_{3,t}\beta\frac{1}{1+\pi_{t+1}}\frac{C_t}{C_{t+1}} + \eta_{4,t} = 0 \tag{1.10}$$

$$\eta_{4,t}I_t = 0 \tag{1.11}$$

$$\eta_{4,t} \geqslant 0 \tag{1.12}$$

$$I_t \geqslant 0 \tag{1.13}$$

Where (1.11)-(1.13) are the conditions for the non-negativity constraint on interest rates. When the ZLB is not binding and $I_t \geqslant 0$, the Lagrange multiplier $\eta_{4,t}$ becomes zero by the Kuhn-Tucker condition in equation (1.11) and interest rates will be determined by the rest of the equations. When the ZLB is binding and $I_t = 0$, the interest rate is set to zero and it remains at this level until $\eta_{4,t}$ becomes zero. The system can be transformed with steady state values, taking the normalization $Z_t = 1$:

$$(1+\tau)(1-\theta) + \theta C v_L(L) + (\beta - 1)\kappa\pi(1+\pi) = 0 \tag{1.14}$$

$$L = C\left(1 + \frac{\kappa}{2}\pi^2\right) \tag{1.15}$$

$$\pi = \beta - 1 \tag{1.16}$$

$$\frac{1}{C} - \eta_1 \left(1 + \frac{\kappa}{2}\pi^2\right) + \eta_2 \theta v_L(L) + \eta_3 (1-\beta) \frac{1}{1+\pi} \frac{1}{C} = 0 \tag{1.17}$$

$$-v_L(L) + \eta_1 + \eta_2 \theta C v_{LL}(L) = 0 \tag{1.18}$$

$$\eta_1 \kappa C \pi + \eta_3 \frac{1}{(1+\pi)^2} = 0 \tag{1.19}$$

$$-\eta_3 \frac{1}{(1+I)^2} + \eta_4 = 0 \tag{1.20}$$

$$I \eta_4 = 0 \tag{1.21}$$

$$\eta_4 \geqslant 0 \ , \ I \geqslant 0 \tag{1.22}$$

For equation (1.20) to hold, taking into account (1.21), we thus have two possible cases:

Case 1: $\eta_4 = 0$, $I > 0$, the ZLB doesn't bind. From equation (1.20) we get that $\eta_3 = 0$ and from equation (1.19) we will obtain that the only feasible solution is $\pi = 0$, as $\eta_1 = 0$ gives an impossible solution. This corresponds to the result from the basic DSGE model with no ZLB where optimal inflation is 0, since there are no benefits associated with non-negative inflation.

Case 2: $\eta_4 > 0$, $I = 0$, the ZLB binds. The system becomes, after substituting $\pi = \beta - 1$:

$$(1+\tau)(1-\theta) + \theta C v_L(L) + (\beta-1)^2 \kappa \beta = 0$$

$$L = C \left[1 + \frac{\kappa}{2}(\beta-1)^2\right]$$

$$1 - \eta_1 C \left[1 + \frac{\kappa}{2}(\beta-1)^2\right] + \eta_2 \theta C v_L(L) + \eta_3 (1-\beta) \frac{1}{\beta} = 0$$

$$-v_L(L) + \eta_1 + \eta_2 \theta C v_{LL}(L) = 0$$

$$\eta_3 = \eta_4 = \beta^2 \kappa \eta_1 C (1-\beta)$$

$$\pi = \beta - 1$$

$$I = 0, \ \eta_4 > 0$$

Since the equilibrium condition at the steady state here is $\eta_3 = \eta_4 = \beta^2 \kappa \eta_1 C(1-\beta)$, we can conclude that the only way for this to hold is $\beta > 1$. This is because η_1, being the Lagrange multiplier on the resource constraint, is negative, the intuition being that the shadow value of relaxing the resource constraint is positive. Our model thus provides no clear solution, because β can't exceed one, so the binding ZLB can't be an equilibrium of the Ramsey problem, but it does provide the key intuition: a shock to the discount factor can make the ZLB bind.

Consequently, this simple DSGE framework without trend inflation is ill-suited to provide an answer to the question of optimal inflation at the ZLB, as it provides only two answers: zero or indeterminacy. It will prove much more relevant in the next sections to include positive steady-state inflation, log-linearize around it and solve the dynamic problem.

3.2 Inflation and Welfare in a New Keynesian DSGE Model

Since we are concerned with the question of inflation rate optimality in a context of binding ZLB, we must mention here the approach and results of Billi (2009), who estimated a New Keynesian DSGE with Calvo (1983) pricing to derive this measure. The model focuses on the policymaker's welfare maximization problem:

$$\max_{\pi_t, x_t, i_t} E_0 \sum_{t=0}^{\infty} \beta^t [(\pi_t - \gamma\pi_{t-1})^2 + \lambda x_t^2] \tag{2.1}$$

s.t.

$$\pi_t - \gamma\pi_{t-1} = \beta E_t(\pi_{t+1} - \gamma\pi_t) + \kappa x_t + u_t \tag{2.2}$$

$$x_t = E_t x_{t+1} - \varphi(i_t - E_t \pi_{t+1} - r_t) \tag{2.3}$$

$$u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_{ut} \tag{2.4}$$

$$r_t = (1-\rho_r)\bar{r} + \rho_r r_{t-1} + \sigma_r \varepsilon_{rt} \tag{2.5}$$

$$i_t \geqslant 0 \tag{2.6}$$

Here, (2.1) expresses the policymaker's objective, which has a quadratic form both in output gap, x_t and the unanticipated component of price changes, given by $\pi_t - \gamma\pi_{t-1}$, where γ is the degree of indexation of prices that are not Calvo-optimized. β is, as before, the subjective discount factor and λ is the weight assigned to the output stability objective. (2.2) describes the optimal behavior of firms under price-setting, with κ the slope parameter and u_t a mark-up shock. (2.3) is the inter-temporal Euler equation, where φ is the inter-temporal elasticity of substitution and r_t the real rate of interest shock. (2.4) gives the law of motion for the exogenous mark-up shock, which is AR(1) with autoregressive coefficient ρ_u and innovation $\sigma_u\varepsilon_{ut}$ which is *iid*, with mean zero and positive standard deviation σ_u. (2.5) gives the law of motion for the real-rate shock, which is also AR(1) with autoregressive coefficient ρ_r and innovation $\sigma_r\varepsilon_{rt}$ which is *iid*, with mean zero and positive standard deviation σ_r. Just as in the model we solved previously, (2.6) represents the ZLB on nominal interest rates.

The model is solved with Kuhn-Tucker conditions, calibrated and estimated. The problem is written:

$$\begin{aligned}\max_{\pi_t, x_t, i_t} \min_{\eta_{1,t}, \eta_{2,t}} E_0 \sum_{t=0}^{\infty} \beta^t \{ & -(\pi_t - \gamma\pi_{t-1})^2 - \lambda x_t^2 + \\ & + \eta_{1,t}[(1+\beta\gamma)\pi_t - \gamma\pi_{t-1} - \kappa x_t - u_t] - \\ & - \eta_{1,t-1}\pi_t + \\ & + \eta_{2,t}[-x_t - \varphi(i_t - r_t^n)] + \\ & + \eta_{2,t-1}\beta^{-1}(x_t + \varphi\pi_t)\}\end{aligned}$$

s.t.

$$u_t = \rho_u u_{t-1} + \sigma_u\varepsilon_{ut}$$

$$r_t = (1-\rho_r)\bar{r} + \rho_r r_{t-1} + \sigma_r\varepsilon_{rt}$$

$$i_t \geqslant 0$$

where $\eta_{1,t}, \eta_{2,t}$ are the Lagrange multipliers associated to constraints (2.2) and (2.3). The Kuhn-Tucker conditions are:

$$\pi_t - \gamma\pi_{t-1} = \beta E_t(\pi_{t+1} - \gamma\pi_t) + \kappa x_t + u_t$$

$$x_t = E_t x_{t+1} - \varphi(i_t - E_t \pi_{t+1} - r_t)$$

$$\pi_t : -2(\pi_t - \gamma \pi_{t-1}) + (1 + \beta\gamma)\eta_{1,t} - \eta_{1,t-1} + \beta^{-1}\varphi\eta_{2,t-1} = 0$$

$$x_t : -2\lambda x_t - \kappa\eta_{1,t} - \eta_{2,t} + \beta^{-1}\eta_{2,t-1} = 0$$

$$i_t : -\varphi\eta_{2,t} i_t = 0,\ \eta_{2,t} \geqslant 0,\ i_t \geqslant 0$$

The last equation imposes similar conditions as in the previous section: either the Lagrange multiplier on the Euler equation is zero or nominal interest rates are zero, and the ZLB binds.

After solving the nonlinear system, calibrating and estimating the model, its predictions for optimal inflation range from 0.2 % to 0.7 %, depending directly on the amplitude of the two shocks. With model uncertainty added to this framework via 'worst case shocks' in the planner's objective function and with the biggest amplitude of monetary and mark-up shocks, optimal inflation is found to be 1.4 %. The mechanism at work here is the trade-off between the insurance against hitting the ZLB provided by higher inflation and the cost of this inflation to the economy. When uncertainty is high, it would be optimal to raise inflation and interest rates as much as possible, but since this is costly, it is not optimal to reach a value that would fully insure against the ZLB (which would be 4% according to the FRB/US forecasting model).

3.3 New Keynesian DSGE Models with Trend Inflation

We use here a small-scale NK DSGE model to evaluate the costs of the ZLB in terms of output lost and to discuss the implications of directly modeling positive steady-state inflation for the welfare analysis. We extend the demonstrative model used in the previous section to include, along a representative household, a representative finished goods producing firm, a continuum of intermediate-goods producing firms and a monetary authority. As we will explain, the NK framework makes use of habit formation in the household's preferences and a Rotemberg pricing formula as before, enhanced this time with partial indexation.

The **household**'s utility function takes the form

$$E_0 \sum_{t=0}^{\infty} \beta^t a_t [ln(C_t - \gamma C_{t-1}) + ln(\frac{M_t}{P_t}) - L_t] \tag{3.1}$$

and its budget constraint is

$$\frac{M_{t-1} + W_t L_t + B_{t-1} + T_t + D_t}{P_t} \geqslant C_t + \frac{M_t}{P_t} + \frac{B_t}{r_t P_t} \tag{3.2}$$

where β is the discount factor, γ is the habit formation parameter, a_t is a preference shock, C_t represents the units of good consumed, P_t the nominal price of this good purchased from the finished-good consuming firm, M_t the money holdings, L_t the units of labor which is rewarded with the wage W_t, B_t the bonds owned by the household, D_t the payments from the dividend on these bonds, T_t a lump-sum money transfer to the household, and, finally, r_t is the nominal interest rate. The model is a rather standard one, the two more particular features that may be pointed out are the habit formation which points to backward-looking behavior in consumption and the preference shock which is assumed to follow an AR(1) process of the following form, where ε_{at} is serially uncorrelated, normally distributed with mean zero and standard deviation σ_a:

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}$$

The household maximizes its utility choosing C_t, L_t, M_t and B_t in every period, and this yields the following first order conditions:

$$\frac{a_t}{C_t - \gamma Ct - 1} - \beta\gamma E_t \left(\frac{a_{t+1}}{C_{t+1} - \gamma C_t} \right) = \zeta_t \tag{3.3}$$

$$a_t = \zeta_t \frac{W_t}{P_t} \tag{3.4}$$

$$\beta r_t E_t \frac{\zeta_{t+1} P_t}{P_{t+1}} = \zeta_t \tag{3.5}$$

$$\frac{M_t}{P_t} = \frac{a_t}{\zeta_t} \frac{r_t}{(r_t - 1)} \tag{3.6}$$

where $\zeta_t > 0$ is the Lagrange multiplier on the budget constraint.

The **representative finished goods-producing firm** uses a quantity $Y_t(i)$ of each intermediate goods, which is purchased at price $P_t(i)$ to manufacture Y_t units of the finished good according to the technology:

$$\left[\int_0^1 Y_t(i)^{\frac{\theta_t-1}{\theta_t}} di\right]^{\frac{\theta_t}{\theta_t-1}} \geqslant Y_t \tag{3.7}$$

where θ_t is a cost-push shock on the firm's desired mark-up which follows an AR(1) process in which the serially uncorrelated innovation $\varepsilon_{\theta t}$ is normally distributed with mean zero and standard deviation σ_θ:

$$\ln(\theta_t) = (1-\rho_\theta)\ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_{\theta t}$$

Thus, in each period the finished goods-producing firm chooses the quantity of intermediate goods it purchases so that it maximizes profits:

$$\max_{Y_t(i)} P_t \left[\int_0^1 Y_t(i)^{\frac{\theta_t-1}{\theta_t}} di\right]^{\frac{\theta_t}{\theta_t-1}} - \int_0^1 P_t(i)Y_t(i)di$$

The first order conditions for this problem yield:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} Y_t \tag{3.8}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t} di\right]^{\frac{1}{1-\theta_t}} \tag{3.9}$$

The **representative intermediate goods-producing firm** manufactures the $Y_t(i)$ units of intermediate goods using the labor of the household, which we called L_t according to a technology:

$$Z_t L_t(i) \geqslant Y_t(i) \tag{3.10}$$

where the aggregate technology shock is characterized by a random walk with drift and serially uncorrelated innovation with mean zero and standard deviation σ_z :

$$ln(Z_t) = ln(z) + ln(Z_{t-1}) + \varepsilon_{zt}$$

In a monopolistically competitive market, the intermediate goods-producing firm chooses its nominal price so that it can meet the demand of the finished goods-producing firm at that price. Like in the previous model, the firm faces a quadratic cost of adjusting its nominal price, as in Rotemberg (1982). The cost is thus given by:

$$\frac{\kappa}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}P_{t-1}(i)}-1\right)^2 Y_t$$

where κ is the magnitude of the price adjustment cost, α gives the nature of price-setting (forward or backward-looking) and, importantly, π is the level of steady-state inflation, which didn't appear before.

In each period, the intermediate goods-producing firm chooses $P_t(i)$ to maximize its market value:

$$\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \Psi_t \tag{3.11}$$

where the value of profits is:

$$\Psi_t = \frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{1-\theta_t} Y_t - \left[\frac{P_t(i)}{P_t}\right]^{-\theta_t}\left(\frac{W_t Y_t}{P_t Z_t}\right) - \frac{\kappa}{2}\left(\frac{P_t(i)}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}P_{t-1}(i)}-1\right)^2 Y_t$$

and $\beta^t\zeta_t$ gives the marginal utility for the household of an additional unit in income from dividends. The first order conditions for this problem are:

$$\begin{aligned}
0 = (1-\theta_t)\left[\frac{P_t(i)}{P_t}\right]^{-\theta_t} + \theta_t\left[\frac{P_t(i)}{P_t}\right]^{-1-\theta_t}\left(\frac{W_t}{P_t Z_t}\right) - \\
-\kappa\left[\frac{P_t(i)}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}P_{t-1}(i)}-1\right]\left[\frac{P_t(i)}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}P_{t-1}(i)}\right] + \\
+\beta\kappa E_t\left\{\left(\frac{\zeta_{t+1}}{\zeta_t}\right)\left[\frac{P_{t+1}(i)}{\pi_t^{\alpha}\pi^{1-\alpha}P_t(i)}-1\right]\left[\frac{P_{t+1}(i)}{\pi_t^{\alpha}\pi^{1-\alpha}P_t(i)}\right]\left[\frac{P_t Y_{t+1}}{P_t(i)Y_t}\right]\right\}
\end{aligned} \tag{3.12}$$

$$Z_t L_t(i) = Y_t(i) \tag{3.13}$$

The **monetary authority - central bank** conducts monetary policy according to a Taylor (1993) rule which responds to fluctuations in inflation, output gap and output growth rate, comparing to their steady state levels π, x and g, according to the coefficients ρ_π, ρ_x and ρ_g. The central bank chooses this steady-state value of inflation, which is its long-run inflation target. The Taylor rule is:

$$\ln\left(\frac{r_t}{r}\right) = \rho_r \ln\left(\frac{r_{t-1}}{r}\right) + \rho_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \rho_x \ln\left(\frac{x_t}{x}\right) + \rho_g \ln\left(\frac{g_t}{g}\right) + \varepsilon_{rt} \quad (3.14)$$

There is also interest rate smoothing, the central bank taking into account the previous levels of the interest rate when setting the current one. As in the case of all the previous shocks, the monetary policy shock ε_{rt} is normally distributed with mean zero and standard deviation σ_r.

Having defined the model, we now proceed to re-writing the system in symmetric equilibrium, where intermediate goods-producing firms make the same decisions and the market clearing conditions for money and bonds hold. In particular, this means that $L_t(i) = L_t$, $Y_t(i) = Y_t$, $P_t(i) = P_t$, $M_t = M_{t-1} + T_t$ and $B_t = B_{t-1} = 0$. The system becomes:

$$Y_t = C_t + \frac{\kappa}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}} - 1\right)^2 Y_t \quad (3.15)$$

$$\zeta_t = \frac{a_t}{C_t - \gamma Ct - 1} - \beta\gamma E_t\left(\frac{a_{t+1}}{C_{t+1} - \gamma C_t}\right) \quad (3.16)$$

$$\zeta_t = \beta r_t E_t\left(\frac{\zeta_{t+1}}{\pi_{t+1}}\right) \quad (3.17)$$

$$\begin{aligned}\theta_t - 1 = \theta_t\left(\frac{a_t}{\zeta_t Z_t}\right) - \kappa\left(\frac{\pi_t}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}} - 1\right)\left(\frac{\pi_t}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}}\right) + \\ + \beta\kappa E_t\left[\left(\frac{\zeta_{t+1}Y_{t+1}}{\zeta_t Y_t}\right)\left(\frac{\pi_{t+1}}{\pi_t^{\alpha}\pi^{1-\alpha}} - 1\right)\left(\frac{\pi_{t+1}}{\pi_t^{\alpha}\pi^{1-\alpha}}\right)\right]\end{aligned} \quad (3.18)$$

$$\ln\left(\frac{r_t}{r}\right) = \rho_r \ln\left(\frac{r_{t-1}}{r}\right) + \rho_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \rho_x \ln\left(\frac{x_t}{x}\right) + \rho_g \ln\left(\frac{g_t}{g}\right) + \varepsilon_{rt} \quad (3.19)$$

$$\frac{1}{Z_t} = \frac{1}{Q_t - \gamma Q_{t-1}} - \beta\gamma E_t\left[\left(\frac{a_{t+1}}{a_t}\right)\left(\frac{1}{Q_{t+1} - \gamma Q_t}\right)\right] \quad (3.20)$$

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at} \quad (3.21)$$

$$\ln(\theta_t) = (1-\rho_\theta)\ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_{\theta t} \tag{3.22}$$

$$ln(Z_t) = ln(z) + ln(Z_{t-1}) + \varepsilon_{zt} \tag{3.23}$$

$$x_t = \frac{Y_t}{Q_t} \tag{3.24}$$

$$g_t = \frac{Y_t}{Y_{t-1}} \tag{3.25}$$

where (3.20) is en equation that needs to hold for the efficient level of output Q_t, (3.21)-(3.23) are the processes followed by the three shocks except monetary policy, (3.24) gives the level of the output gap in relation to its efficient level and (3.25) gives the growth rate of output. In this system with eleven variables and equations, some will have unit roots from the random-walk technology shock (3.23). We need to express equilibrium conditions in terms of stationary variables, so we proceed to detrending the variables: $y_t = Y_t/Z_t$, $c_t = C_t/Z_t$, $q_t = Q_t/Z_t$, $\lambda_t = Z_t\zeta_t$ and $z_t = Z_t/Z_{t-1}$. The system, now stationary, is written:

$$y_t = c_t + \frac{\kappa}{2}\left(\frac{\pi_t}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}} - 1\right)^2 y_t$$

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}$$

$$\lambda_t = \frac{a_t z_t}{z_t c_t - \gamma c_{t-1}} - \beta\gamma E_t\left(\frac{a_{t+1}}{z_{t+1}c_{t+1} - \gamma c_t}\right)$$

$$\lambda_t = \beta r_t E_t(\lambda_{t+1}/z_{t+1}\pi_{t+1})$$

$$\ln(\theta_t) = (1-\rho_\theta)\ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \varepsilon_{\theta t}$$

$$\ln(Z_t) = \ln(z) + \ln(Z_{t-1}) + \varepsilon_{zt}$$

$$\theta_t - 1 = \frac{\theta_t a_t}{\lambda_t} - \kappa\left(\frac{\pi_t}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}} - 1\right)\left(\frac{\pi_t}{\pi_{t-1}^{\alpha}\pi^{1-\alpha}}\right) + \\ + \beta\kappa E_t\left[\left(\frac{\lambda_{t+1}y_{t+1}}{\lambda_t y_t}\right)\left(\frac{\pi_{t+1}}{\pi_t^{\alpha}\pi^{1-\alpha}} - 1\right)\left(\frac{\pi_{t+1}}{\pi_t^{\alpha}\pi^{1-\alpha}}\right)\right]$$

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_\pi \ln(\pi_t/\pi) + \rho_x \ln(x_t/x) + \rho_g \ln(g_t/g) + \varepsilon_{rt}$$

$$g_t = (y_t/y_{t-1})z_t$$

$$1 = \frac{z_t}{z_t q_t - \gamma q_{t-1}} - \beta\gamma E_t \left[\left(\frac{a_{t+1}}{a_t} \right) \left(\frac{1}{z_{t+1}q_{t+1} - \gamma q_t} \right) \right]$$

$$x_t = y_t/q_t$$

In order to use the model for analyzing the response of the economy to one of the shocks, we will log-linearize around a steady state in which inflation is not zero and is exogenously determined. After applying first-order Taylor approximations, we get the final system:

$$\hat{\lambda}_t = \hat{r}_t + E_t\hat{\lambda}_{t+1} - E_t\hat{\pi}_{t+1} \tag{3.26}$$

$$(1 + \beta\alpha)\hat{\pi}_t = \alpha\hat{\pi}_{t-1} + \beta E_t\hat{\pi}_{t+1} - \psi\hat{\lambda}_t + \psi\hat{a}_t + \hat{e}_t \tag{3.27}$$

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + \rho_\pi\hat{\pi}_t + \rho_x\hat{x}_t + \rho_g\hat{g}_t + \varepsilon_{rt} \tag{3.28}$$

$$\begin{aligned}(z - \beta\gamma)(z - \gamma)\hat{\lambda}_t =& \gamma z\hat{y}_{t-1} - (z^2 + \beta\gamma^2)\hat{y}_t + \beta\gamma z E_t\hat{y}_{t+1} \\ &+ (z - \beta\gamma\rho_a)(z - \gamma)\hat{a}_t - \gamma z\hat{z}_t\end{aligned} \tag{3.29}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{at} \tag{3.30}$$

$$\hat{e}_t = \rho_e\hat{e}_{t-1} + \varepsilon_{et} \tag{3.31}$$

$$\hat{z}_t = \varepsilon_{zt} \tag{3.32}$$

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{3.33}$$

$$0 = \gamma z\hat{q}_{t-1} - (z^2 + \beta\gamma^2)\hat{q}_t + \beta\gamma z E_t\hat{q}_{t+1} + \beta\gamma(z - \gamma)(1 - \rho_a)\hat{a}_t - \gamma z\hat{z}_t \tag{3.34}$$

$$\hat{x}_t = \hat{y}_t - \hat{q}_t \tag{3.35}$$

A series of transformations have been applied here. Since $\hat{c}_t = \hat{y}_t$, $\hat{c}_t$ has been dropped from the system. The cost-push shock θ_t is normalized as $\hat{e}_t = -\theta_t \frac{1}{\kappa}$ while $\rho_\theta = \rho_e$. Finally, for convenience, $\psi = \frac{\theta-1}{\gamma}$.

In this system, (3.26) is the New Keynesian IS curve, which links past, present and expected future output to the real interest rate, (3.27) is the New Keynesian Phillips curve with backward and forward-looking components given by α and (3.28) is the Taylor rule.

As for the rest of the equations: (3.29) gives the marginal utility of consumption in terms of past, present and expected output, (3.30)-(3.32) are the laws of motion for preference, cost-push and technology shocks, (3.33) gives the growth rate of output, (3.34) the efficient level of output and (3.35) the output gap.

The model used here follows Canova (2009), focusing on the three equations (3.26)-(3.28) explained before, describing the optimizing behavior of representative households (NK IS curve), the optimizing behavior of monopolistic firms (NK Phillips curve) and how the monetary authority sets short-term interest rates in response to relevant variables and their deviation from the steady state. While the characteristics of the model are relatively standard, here they match the framework used by Ireland(2011).

Blanchard and Kahn (1980) introduced a method for solving such systems of equations. This consists of re-writing the system in the form of a state-space econometric model:

$$A \begin{bmatrix} x_{t+1} \\ E_t y_{t+1} \end{bmatrix} = B \begin{bmatrix} x_t \\ y_t \end{bmatrix} + C\varepsilon_t$$

where x_t is an $(n \times 1)$ vector of state (predetermined) variables, y_t is an $(m \times 1)$ vector of control (or jump) (non-predetermined) variables, and ε_t is an $(k \times 1)$ vector of stochastic shocks at time t. A and B are $(n+m)\times(n+m)$ matrices; C is an $((n+m) \times k)$ matrix. The difference between state and control variables is that the values of the state variables at time $t+1$ do not depend on the $t+1$ shocks; while the values of the control variables depend on them. We re-write our system of equations as:

$$AE_t s^0_{t+1} = Bs^0_t + C\xi_t$$

and we will have four state variables, six control variables and 4 shocks, such that:

$$s_t^0 = \begin{bmatrix} \hat{y}_{t-1} & \hat{\pi}_{t-1} & \hat{r}_{t-1} & \hat{q}_{t-1} & \hat{x}_t & \hat{g}_t & \hat{\lambda}_t & \hat{y}_t & \hat{\pi}_t & \hat{q}_t \end{bmatrix}'$$

$$s_{t+1}^0 = \begin{bmatrix} \hat{y}_t & \hat{\pi}_t & \hat{r}_t & \hat{q}_t & \hat{x}_{t+1} & \hat{g}_{t+1} & \hat{\lambda}_{t+1} & \hat{y}_{t+1} & \hat{\pi}_{t+1} & \hat{q}_{t+1} \end{bmatrix}'$$

$$\xi_t = \begin{bmatrix} \hat{a}_t & \hat{e}_t & \hat{z}_t & \varepsilon_{rt} \end{bmatrix}'$$

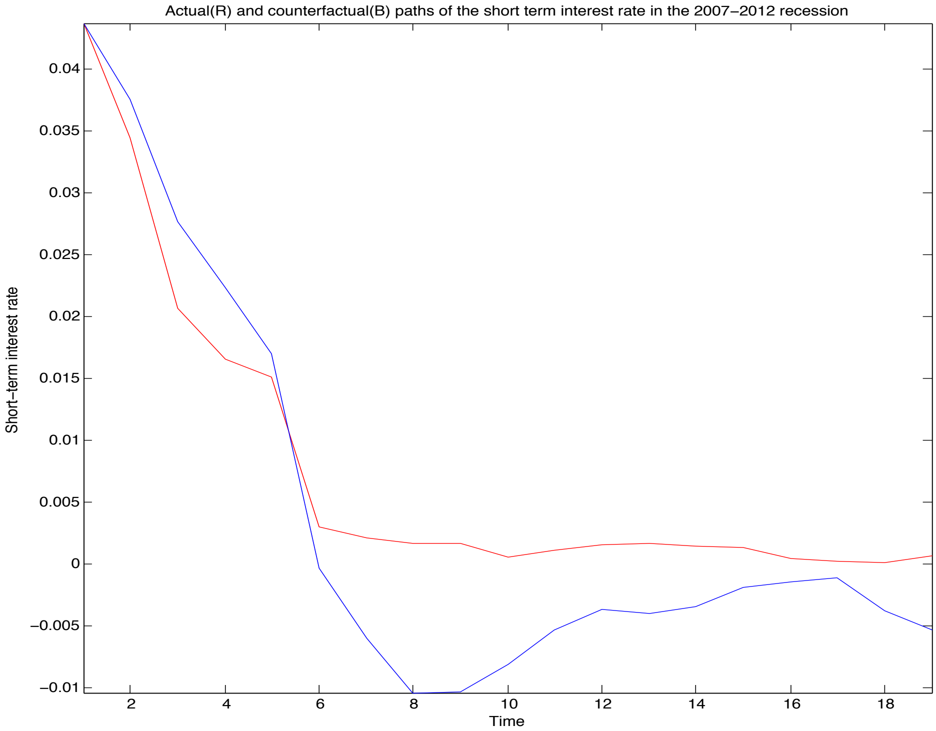
The procedure used to derive the solution of this model is further detailed in the Annex A of the paper. The solution links the behavior of the observable variables - output growth rate, inflation rate and short-term nominal interest rate - to a vector of unobserved state variables, four of which are the exogenous shocks. We can use Kalman filtering and smoothing after having written the solution as a state-space econometric model and this will allow us to obtain maximum likelihood estimates of the model's structural parameters. The computational steps that lead to this are presented in more detail in Annex B. Using the estimates, we are then able to run a policy counterfactual which compares the actual paths for the three observable variables with the counterfactual path of the same variables given by the smoothed estimates. If we run this counterfactual without imposing any restriction on the model, this would only serve to compare the model predictions with reality, and we would actually see that the fit between the two is almost perfect. But our goal here is to see how the ZLB impacts the economy, so we will compare the actual path for the three state variables to the counterfactual path obtained when three out of the four shocks are allowed to impact; more precisely, we will zero out the monetary policy shock.

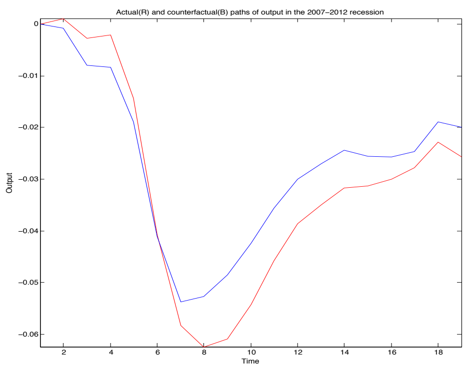
The simulation is run on US quarterly data, running from 1983:Q1 to 2012:Q1, taken from the Federal Reserve Bank of St. Louis' FRED 2 Database. The state variables are defined as follows:

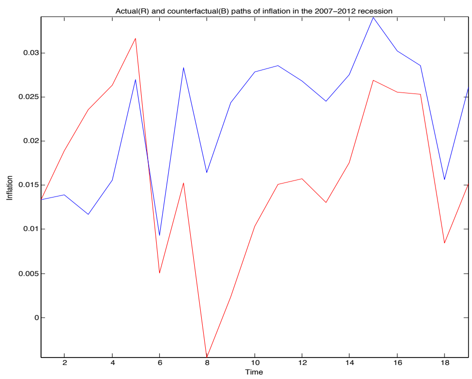
- Outupt growth: seasonally-adjusted, quarter-to-quarter changes in natural logarithm of real GDP in chained 2005 dollars (GDPC96), divided by the civilian non institutional population ages 16 and over (CNP16OV) to get per-capita measures.
- Inflation rate: seasonally-adjusted, quarter-to-quarter changes in natural logarithm of the GDP deflator (GDPDEF)
- Short-term nominal interest rate: quarterly averages on the three-month US Treasury bill daily rate (TB3MS), coverted to quarterly yield to maturity

The results of these simulations are presented in the form of three graphs on the next page, which plot actual and counterfactual data for the time interval 2007:Q1-2012:Q1. In the biggest graph we can see that had the ZLB not started to bind in 2009, the US Federal Reserve would have further decreased the short-term nominal interest rate by 100 basis points (that is, by 1%), according to the Taylor rule in our model. This would have allowed a more rapid recovery for output, as shown in the smaller graph on the lower-left side. Furthermore, we notice that the ZLB still binds, as the counterfactual of interest rate is still under the actual value in 2012:Q1. Inflation, in the lower-right side, would have had slightly higher levels, but by the end of the dataset, it would have been close to its actual value. Consequently, estimating a model such as the one presented in this section seems to provide some support to the view that inflation targets can be augmented by 1% to prevent hitting the ZLB and thus achieving a sub-optimal recovery path. It remains to be seen if welfare analysis also lends supports this view.

Results of Counterfactual Simulations for the State Variables







3.4 Inflation and Welfare in a New Keynesian DSGE Model with Trend Inflation

While the results of these simulations and counterfactuals illustrate the extent to which conventional monetary policy was and still is constrained by the ZLB, they don't address the issue of optimality of the inflation rate. Welfare considerations however are key to understanding if a permanently higher inflation target is mandated as an insurance policy against the non-negativity constraint on nominal interest rates. It is possible to use the NK DSGE framework with log-linearization around a steady state which need not be zero to estimate optimal inflation. Indeed, this is the focus of very recent research in the field of monetary policy. Coibion, Gorodnichenko, Wieland (2012) build a model which is almost identical to the one used in the previous section of this paper, while also attaching a welfare function to it. We briefly present this model in what follows.

The utility-maximization function for the representative household takes the form:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ln(C_t - \gamma g_t^A C_{t-1}) - \frac{\eta}{\eta+1} \int_0^1 N_t(i)^{1+\frac{1}{\eta}} di \right] \tag{4.1}$$

and the budget constraint is:

$$\int_0^1 \frac{N_t(i) W_t(i)}{P_t} di + \frac{B_{t-1} q_{t-1} R_{t-1}}{P_t} + T_t \geqslant C_t + \frac{B_t}{P_t} \tag{4.2}$$

If we compare these two equations with equations (3.1) and (3.2) in the previous section, we can see they have similar configurations and variables: β is the discount factor, γ is the habit formation parameter, C_t represents the units of final good consumed, P_t the nominal price of this good purchased from the finished-good consuming firm, N_t the units of labor supplied to industry i, which gets rewarded with the wage W_t, B_t the bonds owned by the household, T_t a lump-sum money transfer to the household, and, finally, R_t is the nominal interest rate. What is new here is g_t^A which is the growth rate of technology, η which is the Frisch labor supply elasticity, q_t is a risk-premium shock and ξ_t will be the shadow value of wealth.

The first-order conditions of this maximization problem are:

$$\frac{1}{(C_t - \gamma g_t^A C_{t-1})} - \frac{\beta \gamma E_t g_{t+1}^A}{(C_{t+1} - \gamma g_{t+1}^A C_t)} = \xi_t \tag{4.3}$$

$$N_t(i)^{\frac{1}{\eta}} = \frac{\xi_t W_t(i)}{P_t} \tag{4.4}$$

$$\frac{\xi_t}{P_t} = \beta E_t \left[\frac{\xi_{t+1} q_t R_t}{P_{t+1}} \right] \tag{4.5}$$

These are equivalent to equations (3.3) - (3.5). Similarities continue, as the production of the final good is done by a perfectly competitive sector which aggregates a continuum of intermediate goods as per:

$$\left[\int_0^1 Y_t(i)^{\frac{\theta_t - 1}{\theta_t}} di \right]^{\frac{\theta_t}{\theta_t - 1}} \geqslant Y_t \tag{4.6}$$

where θ_t is the elasticity of substitution across intermediate goods and Y_t the final good resulting fro the aggregation. This corresponds perfectly to equation (3.7). The first order conditions for this problem will again give the demand curve for goods of the intermediate sector i and the aggregate price level, as in equations (3.8) and (3.9):

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t \tag{4.7}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_t} di \right]^{\frac{1}{1-\theta_t}} \tag{4.8}$$

A monopolist manufactures the $Y_t(i)$ units of intermediate goods using the labor of the household, which we called N_t and technology Z_t as in equation (3.10):

$$Z_t N_t(i) \geqslant Y_t(i) \tag{4.9}$$

There is, however, another difference between these two models, as price stickiness is here modeled as in Calvo (1983), with λ the probability that firms will not be able to reoptimize prices each period. Firms that don't reoptimize, index prices to steady-state inflation $\bar{\Pi}$, with the degree of indexation ω. The optimal reset price is denoted $P^o(i)$.

The firm's profit maximization problem is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \lambda^t [Y_t(i) P_t^o(i) \bar{\Pi}^{t\omega} - W_t(i) N_t(i)] \tag{4.10}$$

and the price dynamics are given by:

$$P_t^{1-\theta} = (1-\lambda)(P_t^o)^{1-\theta} + \lambda P_{t-1}^{1-\theta}\bar{\Pi}^{\omega(1-\theta)} \tag{4.11}$$

There is government consumption in this economy, so the market clearing condition is $Y_t = C_t + G_t$, whereas aggregate labour is obtained by aggregation: $N_t = \left[\int_0^1 N_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$

The model is log-linearized around a steady-state with positive inflation. From a notation viewpoint, variables with bar denote steady-state values, lowercase letters denote the log of the variable, and hats denote log-deviations from the steady-state. The risk-premium, government and Phillips Curve shocks follow AR(1) processes, whereas the technology follows a random walk with drift, as in the previous section.

The system becomes:

$$\hat{\xi}_t = E_t(\hat{\xi}_{t+1} + \hat{r}_t - \hat{\pi}_{t+1} + \hat{q}_t) \tag{4.12}$$

$$\hat{y}_t = \bar{c}_y\hat{c}_t + \bar{g}_y\hat{g}_t \tag{4.13}$$

$$\hat{y}_t = \hat{n}_t \tag{4.14}$$

where (4.12) is the consumption Euler equation, (4.13) is the goods market clearing condition and (4.14) is obtained after integrating across production functions. The effects of allowing for positive steady-state inflation can be best seen in the following equations:

$$\bar{X}^{\frac{\eta+1}{\eta}} = \frac{1-\lambda\beta\bar{\Pi}^{\frac{(1-\omega)\theta(\eta+1)}{\eta}}}{1-\lambda\beta\bar{\Pi}^{(1-\omega)(\theta-1)}}\left(\frac{1-\lambda}{1-\lambda\bar{\Pi}^{(1-\omega)(\theta-1)}}\right)^{\frac{\eta+\theta}{\eta(\theta-1)}} \tag{4.15}$$

This gives the steady-state level of the output gap, which is equal to one when steady-state inflation is zero ($\bar{\Pi} = 1$) or when the degree of price indexation ω is 1. The equation shows there is a non-linear relationship between the steady-state levels of inflation and output. $\bar{X}$ is increasing with trend inflation when this has low positive values, but starts to fall for larger values.

$$\frac{\bar{P}^o}{P} = \left(\frac{1-\lambda}{1-\lambda \bar{\Pi}^{(1-\omega)(\theta-1)}} \right)^{\frac{1}{\theta-1}} \tag{4.16}$$

Which is the relationship between inflation and the re-optimizing price: inflation is less sensitive to changes in re-optimizing price as trend inflation increases.

Finally, the log-linearized optimal reset price equation shows that higher trend inflation increases the weight put by firms on future output and inflation when setting new prices, and adds a term which gives some weight to future differences between output growth and interest rates. The result is that trend inflation makes price-setting decisions more forward-looking, as reset prices become more responsive to current shocks.

The monetary authority follows a Taylor rule, which closes the model:

$$\begin{aligned} \hat{r}_t^* &= \rho_1 \hat{r}_{t-1}^* + \rho_2 \hat{r}_{t-2}^* + \\ &+ (1-\rho_1-\rho_2)[\phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y*) + \phi_g y(gy_t - gy*) + \phi_p(p_t - p*)] + \\ &+ \varepsilon_t^r \end{aligned} \tag{4.17}$$

The central bank smooths interest according to ρ_1, ρ_2 two periods back and places weights $\phi_\pi, \phi_y, \phi_{gy}, \phi_p$ on deviations from steady-state of inflation, the output gap, the output growth rate and the price level. ε_t^r is a policy shock and the nominal interest rate is bounded by zero.

As we can deduce from the log-linearized system (4.12)-(4.17), there are three costs for inflation in this model:

- Steady-state effects from positive trend inflation, caused by an increase in steady-state dispersion prices across the board. This leads to inefficient allocations of resources across sectors and a decrease in the steady-state level of output. The cost arises naturally from integration of positive trend inflation in a New Keynesian DSGE model.

- An inflationary shock as well as positive trend inflation create distortions in relative prices. This leads to larger marginal disutility of labour which is costly for firms who have to compensate workers as a result. Consequently, the variance of inflation is costlier for welfare.

- Trend inflation makes price-setting decisions more forward-looking, as the weight put by firms on future output and inflation increases and reset prices become more responsive to current shocks.

The benefit of positive inflation is solely the reduced incidence of a binding ZLB.

In Coibion, Gorodnichenko, Wieland (2012), the impact of these costs and benefits on welfare are quantified through a welfare function derived from a second order approximation of the household utility function. Calibration and estimation for different levels of steady state inflation lead to interesting results. While in the absence of the ZLB optimal inflation is zero, when both costs and benefits of inflation are included in the simulation, a peak level of utility is reached at 1,5% annualized inflation. This level of steady-state inflation is enough to protect the economy from the effect of hitting the ZLB. Reasonable variations in the calibration of parameters in the model have little effect on the optimal rate of inflation. Furthermore, even modifications in the characteristics of the model (e.g. Taylor pricing, including capital or downward nominal wage rigidities) don't significantly affect the optimal rate of inflation, which is typically less than 2% per year. Consequently, New Keynesian DSGE Models don't seem to provide any basis for an increase in the long-run inflation target of central banks, even if the Zero Lower Bound and its effects are accounted for.

4 A New Post-Crisis Monetary Policy?

In the first section of the paper we have used a New Keynesian DSGE framework to verify if a higher permanent inflation target would be welfare improving, given the extremely negative effects of a conventional monetary policy constrained by the Zero Lower Bound. Incorporating positive steady-state inflation into the model increases the optimal inflation rate from zero, but the new value is in any situation below 2%. Using a policy counterfactual applied on US data, we concluded that the Zero Lower Bound did bind and that, had this not been the case, only a further 1% cut in nominal interest rates would have ensured a faster recovery from the crisis.

There are however several caveats to this analysis that need to be mentioned. First, we have focused exclusively on conventional monetary policy as a tool of intervention, abstracting from unconventional monetary policy tools that may be used to fight a binding ZLB. Similarly, our focus has been solely on monetary policy, neglecting the fact that fiscal policy is also quite effective in tackling the effects of the ZLB, as a series of recent contributions in the economic literature have proved. Finally, the New Keynesian models we used only assume one benefit for positive trend inflation: lower incidence of a binding non-negativity constraint on nominal interest rates. If in the long run this may be accurate, higher inflation provides a number of other benefits in the short and medium run.

We now turn to analyzing inflation targeting and, more broadly, monetary policy instruments and objectives and their possible evolution after the Great Recession. Throughout this section, we will address two questions: "Why would we need a higher inflation target?" and "Is targeting a level of inflation enough to ensure proper monetary policy actions?". As opposed to the previous sections, we will focus on short and medium-term inflation targets.

Even if we have chosen previously to discuss higher inflation targets as a way to ensure that the non-negativity constraint on nominal interest rates doesn't bind, there are other reasons to consider this proposal. The following points are relevant:

- Inflation may serve as an adjustment tool, whose purpose would be the re-alignment of relative wages and prices inside a monetary union, among the "core" and the "periphery", thus avoiding the painful and unfeasible option of internal devaluation.
- The debt burden of countries might be alleviated by accepting slightly

higher inflation for a certain, limited amount of time. Highly problematic outcomes of insolvency and default may thus be avoided.

- In some situations, the relative cost of labor compared to a basket of consumption may be considered high by employers. Since nominal wages are highly rigid downwardly, a way to make labor cheaper and thus increase employment would be to raise inflation.

- The process of exiting a severe recession involves constantly robust growth, which might lead to higher levels of inflation than generally accepted by monetary authorities. Central Banks should not take action to curb inflation in this situation, as this might compromise the recovery.

Even if it started with adverse developments in the United States in 2007-2008, the Great Recession continues to negatively affect the global economic activity through its effects on the economies of the Eurozone. It is a generally accepted conclusion by now that these powerful headwinds are not so much a consequence of particularly weak economic fundamentals of European countries, but of the inadequacy of Europe-wide economic institutions, which tend to exacerbate any such flaw.

To illustrate this, we can think of the Eurozone as a basic two-country model: the "core", 3/4 of the entire economy and the "periphery", 1/4 of the entire economy. Inflation in the core and periphery will have similar weights in the overall indicator. Assuming that wages are 20% higher in the periphery than in the core and that we would want to adjust this difference in the next five years, this means that we will need inflation in the core to be 4 percentage points higher than the one in the periphery for each of those years. A mixture of inflation in the core and deflation in the periphery may ensure this spread, but this can be accomplished in several ways. For our purposes, we consider two allocations of the form (inflation core, deflation periphery), namely (4,0) which leads to Eurozone inflation of 3% and (2, -2) which leads to Eurozone inflation of 1%.

Our simplified analysis reflects rather accurately the plight of peripheral Eurozone countries like Spain and Portugal which have been left with a competitiveness problem following monetary integration, that they now need to rapidly correct in their search for more investment and exports. Since they have the euro as currency, they cannot simply resort to currency devaluation, as this is not their decision to make. The allocation (4,0) would produce the needed convergence effect, but it would mean accepting higher, above-target Eurozone inflation of 3%. The second allocation, (2, -2) would achieve the same result, while only requiring 1%, under-target Eurozone inflation. An ultra-conservative central bank, like the ECB is considered to be, would opt for the latter. However, demanding a 2% deflation in the periphery is very unrealistic and very hard to accept socially. It is unrealistic because deflation means reduction in nominal wages and this almost never happens, as wages are downwardly rigid, especially in Europe where the labour market is comparatively less flexible than in other regions. It is painful and hard to accept socially, because instrumenting a decrease of internal demand by the periphery, in search of trade surpluses could lead to sky-rocketing unemployment rates: 29% in Spain, 36% in Portugal and 52% in Greece, as estimated by Artus (2012a). It's hard to see then why this course of action would be preferable, because demanding this sort of sacrifice from any country would almost certainly put it on a path of prolonged recession.

Allowing for modest above-target inflation in the Eurozone, would further generate two positive developments. First, it would allow a measure of reduction in the debt burden of the same peripheral economies that experience low competitiveness, since they are also saddled with significant amounts of sovereign debt, for various reasons. And second, it would act as a disincentive to hoard cash, which is what big corporations and affluent households tend to do in times of crisis. Consequently, money that is now simply stored could rather be directed toward profitable investment, which would provide some much-needed additional stimulus for the economy.

Since the only thing standing in the way of adopting the less harmful allocation is a strict adherence to the 2% inflation target, adopting a higher target could yield obvious welfare-improving results. This is all the more plausible since the downside of temporarily having a slightly higher inflation target is ambiguous at best as we will further discuss.

The tradeoff in the Eurozone between slightly higher inflation and the possibility of growth may be explained by the mandate of the European Central Bank, which is only concerned with price stability and aims for inflation to be close but under 2%. But the reality of the situation illustrates how this objective, which is generally the same for monetary authorities the world over, may become divorced from economic welfare. An economy can indeed experience at the same time higher inflation and subpar growth and this argument has been put forward in support for the proposal to extend central banks' mandates, so that they may put an equal weight on economic performance, measured by the level of employment. The reasoning goes that, if such a mandate is enacted, the monetary authority will not have the possibility to choose to hold its inflation target even when unemployment consequences would be as dire as described above.

Most central banks, including the ECB and the Bank of England, are not concerned with employment rate objectives and monitor exclusively price stability and financial security. The Federal Reserve System of the United States is, however, an exception to this rule, as it has a dual mandate, which includes the maximization of employment along with price stability. In normal times, these two objectives don't collide, because when inflation is high and needs to be lowered through monetary tightening, employment is usually close to full, because the economy is in a boom. But recent developments have challenged the way the Fed understands its role and delivers on it and have highlighted a conflict between its two objectives.

When the Zero Lower Bound binds, the monetary authority loses one of its policy tools, the ability to lower the short-term nominal interest rate to stimulate the economy. But it can still influence the expectations of economic agents regarding future inflation, adopting a stance on long-term interest rates through what is known as forward guidance. The Federal Reserve has, very recently, made use of this tool when publishing a press release on January 25th 2012 with the following statement[3]:

> "To support a stronger economic recovery and to help ensure that inflation, over time, is at levels consistent with the dual mandate, the Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions - including low rates of resource utilization and a subdued outlook for inflation over the medium run - are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014."

By promising to keep low short-term interest rates for at least another two years, the Fed is effectively influencing expectations of long-term interest rates. To the extent to which markets consider this commitment to be credible, the language used should be stimulative, because what it hints is that the Fed would be willing to tolerate above-target inflation for a fixed amount of time, in order to pursue rapid economic growth, even if this means deviating from its normal policy rule. This can also be explained as an attempt to influence the inflation-adjusted interest rate, given by the standard Fisher equation $r = i - \pi^e$. Furthermore, the decision to purchase long-term treasuries in addition to this commitment followed the same logic.

While the decision to take this commitment has been stimulative and is indeed what most economists agree is needed when the ZLB binds, a new problem arose when inflation approached the target level of 2%. This followed

[3]Federal Open Market Committee Press Release on Monetary Policy, available at: http://www.federalreserve.gov/newsevents/press/monetary/20120125a.htm

a series of positive developments in the labor markets which saw unemployment fall to about 8%, better than the initial starting point, but still very far off from event the most conservative estimates of the US full employment rate. Faced with the prospect of overshooting the inflation target, the Fed backtracked on its stimulative policies, effectively saying through members of the Federal Open Market Committee that[4]

> "Doing more at this time could create too much inflation risk, and doing less could risk weakening an already slow expansion and causing an unwelcome disinflation."

This approach is all the more surprising since it was made at the same time when the Fed was projecting that unemployment will remain a bit above 8% throughout the year, with a growth rate of the economy estimated at around 3%. Putting this into perspective, the Fed is thus successful in one of its objectives, namely inflation targeting, but fails quite substantially its second objective, the pursuit of full employment. A contrast arises between the stated policy of the Federal Reserve and the actual policy that it follows, which became apparent through revealed preference.

At the ZLB, the real rate of interest is just the negative rate of inflation, so that any upper bound on inflation sets a lower bound on the real interest rate. If this lower bound is still higher than the real interest rate needed for market-clearing, the dual mandate becomes impossible to implement, and the monetary authority must choose the objective it considers most important. In the case of the Federal Reserve, the maximum monetary easing, within the bounds of the 2% inflation target doesn't seem to provide the stimulus that is needed to engineer a solid and rapid recovery. Consequently, it looks like the Fed is willing to accept prolonged weakness in the labor market rather than allow any overshoot of the inflation target, no matter how transitory. Adding this to a political context that allows no fiscal stimulus and assuming that economic recovery may be based on expectations of monetary easing, we may conclude that the choice made by the Fed amounts in effect to a tightening of policy.

Both accounts described so far in this section seem to point to the fact that 2% inflation targeting is still perceived by most monetary authorities as their paramount objective, and that overshooting this target is never desirable, no matter what the positive effects on the measures of economic activity may be. After discussing the benefits of higher inflation targets, at

[4] Sandra Pianalto interview available at: http://www.bloomberg.com/news/2012-03-01/pianalto-says-she-s-comfortable-with-current-stance-of-monetary-policy.html

least for a fixed amount of time, we now turn to the main objection put forward that deters central banks from allowing any persistent deviations from the currently accepted inflation target:

- Allowing for higher inflation would cause future inflation expectations to become de-anchored and diverge to high, unsustainable levels. Hyperinflation would ensue and the cost of bringing prices down from such high levels would be immense.

This is the argument that was made by former German central banker Karl O. Pöhl who famously stated that inflation is like toothpaste: very easy to get out of the tube, almost impossible to put back in. The risk is that when the inflation target is raised, politicians fail to control prices and this leads to socially destructive economic consequences for savers and investors. Another critique opposed to the possibility of raising the inflation target is along the lines of Issing's (2012) argument, which states that this would imply a loss of credibility for the central bank. Why would people not expect the inflation target to go from 4% to 6% or from 6% to 8% after it was initially changed from 2% to 4%? In other words, how can inflation expectations be anchored in this approach?

It is a well established belief of central monetary authorities that upside and downside risks are not equally important and that the primary concern should always be hitting the inflation target, because, in the end, what a central bank can deliver is mainly price stability and nothing else. Furthermore, there is an understandable risk-aversion for higher-than normal inflation, considering the events of the Volcker years and the major achievement of price stability which the Great Moderation is considered to be. However, there is little in the way of conclusive evidence to support the hypothesis that inflation expectations would suddenly diverge if the target would be modestly overpassed for a limited amount of time, especially given the rather extraordinary economic conditions which are noticeable in developed economies like the US and the Eurozone.

All of the measures which are commonly used to assess inflation expectations: breakevens, the producer price index and equity data, have low values, pointing to sinking expectations (See Annex C). But most importantly, unemployment is still high and wage growth is inexistent; if we accept that a diverging spiral of prices needs the wage mechanism to occur, then there is no evidence of core inflation in the medium run, neither in the US nor the Eurozone (oil price variations could generate temporary spikes in the headline CPI, but these should be treated as transitory).

Central bankers use some version of the Taylor rule to guide their actions, putting different weights on inflation and output growth. Taylor rules act like automatic stabilizers, which are very useful because, once legislated, they allow for immediate action from policy makers. In normal economic circumstances, this is fine: a fall in output is swiftly compensated by monetary easing and an over-heating of the economy is counterbalanced by tightening. But a different approach may be needed when output starts to rise after having reached bottom; in this situation, an automatic trigger that would dampen the recovery is not desirable. Better results can be achieved with some sort of formula flexibility, maybe similar to the recent "7/3 rule" put forward by Evans (2011), which states that the FOMC should only raise rates if unemployment falls below 7% or core inflation rises above 3%.

A much stronger case has to be made by economists and central banks in support of the policy of 2% inflation targeting that is currently being rigidly implemented. While upside risks are very plausible in normal times and the benefits of a higher inflation target are uncertain, situations in which downside risks are overwhelmingly predominant are more ambiguous. It seems hard to believe that inflation expectations could become de-anchored if slightly higher inflation would transparently be tolerated for a limited amount of time in pursuit of full employment. Given the potentially significant benefits of higher inflation for the reduction of unemployment, it may well be that, in lack of evidence to the contrary, the welfare-maximizing level of inflation could be higher in recessions and ZLB situations than in normal times. The chief benefit of having well-anchored inflation expectations is that a limited period of above-target inflation wouldn't disturb the long-run inflation outlook. Central banks may find that they need to take advantage of this.

The credibility of a central bank is thus of paramount importance and the fear of losing it is what stops monetary authorities from engineering a more stimulative policy through temporary overshooting. But the credibility argument can be looked at in another way. If central banks reject any overshooting when economic realities might justify it, they will indeed enjoy very strong credibility, so much so that if they will ever need to create inflation expectations in deflation times, when this is most needed, they might find that they no longer have the ability to do so, as their commitment to future higher inflation will not be believed by the markets.

In discussing monetary policy issues we have focused exclusively on a single objective and a single instrument: inflation targeting through short-term interest rate adjustment. And with good reason, since the consensus among central bankers before the crisis was that simple inflation targeting is the state of the art approach to monetary policy. Only when the ZLB binds, which wasn't expected to happen very often, would unconventional interventions, such as quantitative easing, be acceptable. The reasoning behind this approach was that achieving stable inflation through conventional intervention would suffice to ensure macro stability, or a stable output gap; in other words, the evolution of these two indicators was considered to be closely correlated. This simple, straight-forward way to understand and apply monetary policy lost its credibility after the crisis, as two weaknesses came to attention. First, the evolutions of inflation and output gap need not be correlated, mainly due to downward price and wage rigidities, and second, even if inflation and output are stable, there are other factors - financial instability shocks for instance - which can negatively impact the economy and should thus be closely monitored.

In this context and taking into account that the main central banks are beginning to implement exit strategies from the unconventional policies they are currently undertaking, the question that is being asked is whether a return to pre-crisis monetary policy is warranted or if monetary policy has to be expanded to include more objectives, which could be approached with more instruments. Given that the unique objective of price stability proved insufficient, both in preventing the crisis and in effectively addressing its consequences, a policy of more objectives and instruments seems beneficial, in line with Blanchard's (2012) proposal. While this would be a step back from the neat pre-crisis consensus, it may well prove to be a more effective approach to implement monetary policy in a welfare-maximizing way.

Adding several instruments and objectives to the monetary mix will enable central banks to take account of heterogeneous economic situations, which can't all be effectively addressed using the single-objective, single-instrument approach, as Artus (2012b) pointed out. In the Eurozone, for instance, there are different ways in which financing is done, mainly fixed-rate and floating-rate. In the latter approach, which is widespread in Southern Europe, the interest paid on mortgage loans varies in relation with the benchmark interest rate set by the ECB. It follows that any conventional action undertaken by the ECB will immediately affect the economies of these countries, while in countries that use fixed rates primarily, the effects are more latent. We briefly discuss here possible objectives and instruments that may be considered:

- The level of the short-term interest instrument can be coordinated with both inflation and output gap. As the dynamics of these two indicators seem to become more and more uncorrelated and especially so during crises, central bankers must monitor the level of the output gap just as closely as they do the evolution of inflation as opposed to assuming that inflation stability will automatic lead to output gap stability.

- Given that both inflation and output gap indicators have been stable before the crisis and thus did not indicate underlying problems in the economy, monitoring financial stability through macro prudential means will provide a more complex overview for policymakers. In other words, closely tracking macroeconomic stability should no longer be enough for central banks, as financial stability also concerns them.

- Quite importantly, targeting inflation and output gap through the policy rate and ensuring financial stability through macro prudential control should not be two separate activities, undertaken by two separate entities, as there are a series of correlations between them. For instance, a lower interest rate encourages risk-taking behavior and may require tighter control, whereas higher reserve requirements for banks who provide mortgages might affect residential investment and the total income of the economy.

- Stabilization of the exchange rate may constitute an objective for monetary authorities, as the evolution of this indicator can strongly influence the economic perspectives of a country. To understand why, we can look at balance-of-payments crisis situations, such as in Mexico 1994, South-East Asia 1997 and Argentina 1999. Once a certain value or growth rate of the exchange rate is set as an objective, sterilized intervention by the central bank can be the instrument used to achieve this goal.

- Reasonable credit growth is another issue. At the same time, banks can engage in solid loan activities which serve the productive investments of companies and in mortgage loans which lead to the unsustainable growth of housing prices. Obviously, the social value of the first types of loan is much bigger and a central bank would want to take this into account and adjust its macro prudential policy accordingly. Furthermore, while some countries may experience growth and strong credit conditions, others may experience slow growth caused by tight credit conditions, and these discrepancies may appear amongst countries which belong to a monetary union.

The interplay of all these objectives and instruments and the appropriate values of the targets set are beyond the scope of this paper. But after an evaluation of pre-crisis monetary policy and of monetary policy as it is currently understood, it seems that we are heading from a one-target one-instrument approach to a multi-target multi-instrument approach. The challenge for future research will be to draw a much clearer picture of how such a new monetary policy would look like.

5 Conclusion

In this paper we have looked at the issue of optimal inflation at the Zero Lower Bound, investigating whether or not it is welfare-improving to permanently increase central banks' inflation targets above the current 2% objective. Using a New Keynesian DSGE framework, first with zero steady-state inflation and then with trend inflation, we have found no proof that this would be desirable, as estimates for optimal inflation given by the models are consistent with the current target. The main reason for this is that New Keynesian DSGEs attach several costs but only one benefit to higher trend inflation, namely reduced incidence of hitting the ZLB. As the inflation target rises above a certain threshold, usually between 2% and 3%, costs outweigh this single benefit. We have however shown, using a policy counterfactual on US data, that conventional monetary policy has been restricted by the ZLB during the current crisis and that an additional 1% wiggle room would have been needed to ensure an optimal response by the monetary authority to the severe negative shocks that have occurred.

Switching to an empirical analysis, we find that *temporary* higher inflation targets have considerable benefits: alleviating debt burdens, serving as an adjustment tool in a monetary union or reducing the real cost of labor when unemployment is high (none of which have yet been modeled into a DSGE framework). These benefits provide justification for temporary higher targets or transparent overshooting of the 2% target for a limited period. Furthermore, these considerations justify awarding a double mandate to monetary authorities, who should pursue price stability and low unemployment. In this context, we addressed the case of the Federal Reserve which follows a double mandate, but whose strict adherence to 2% inflation targeting makes it reluctant to allow a strong job-creating and mildly inflationary recovery in the US.

Lastly, we discussed the outlook for post-crisis monetary policy. Rather than going back to the one-target one-instrument strategy of inflation targeting, a consensus seems to be forming around a more complex form of monetary policy that should be better suited to current heterogeneous economic realities. This would include several instruments (e.g. short-term interest rates, unconventional interventions, macro prudential control) and several targets (e.g. inflation, output gap, unemployment, financial stability, exchange rate stability or credit growth).

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A Blanchard-Kahn Solution Method for the NK DGSE Model with Positive Steady-State Inflation

Since the state vector's covariance matrix turns is singular because of the presence of lagged endogenous variables in the state vector, we will use a generalized Schur decomposition. The system becomes:

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta\gamma z & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta\gamma z \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{y}_t \\ \hat{\pi}_t \\ \hat{r}_t \\ \hat{q}_t \\ \hat{x}_{t+1} \\ \hat{g}_{t+1} \\ \hat{\lambda}_{t+1} \\ \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{q}_{t+1}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
-\gamma z & 0 & 0 & 0 & 0 & 0 & (z-\beta\gamma)(z-\gamma) & z^2+\beta\gamma^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & -\alpha & 0 & 0 & 0 & 0 & \psi & 0 & 1+\beta\alpha & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\gamma z & 0 & 0 & 0 & 0 & 0 & z^2+\beta\gamma^2 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_a & 0 & \rho_x & \rho_g & 0 & 0 & \rho_\pi & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \\ \hat{q}_{t-1} \\ \hat{x}_t \\ \hat{g}_t \\ \hat{\lambda}_t \\ \hat{y}_t \\ \hat{\pi}_t \\ \hat{q}_t
\end{bmatrix}
+
$$

$$
\begin{bmatrix}
-(z-\beta\gamma\rho_a)(z-\gamma) & 0 & \gamma z & 0 & \\
0 & 0 & 0 & 0 & \\
-\psi & -1 & 0 & 0 & \\
0 & 0 & -1 & 0 & \\
-\beta\gamma(z-\gamma)(1-\rho_a) & 0 & 0 & \gamma z & 0 \\
0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 1 & \\
0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 &
\end{bmatrix}
\xi_t
$$

Furthermore, we can write as a system of linear expectational difference equations driven by the exogenous shocks:

$$\xi_t = P\xi_{t-1} + \varepsilon_t$$

where

$$P = \begin{bmatrix} \rho_a & 0 & 0 & 0 \\ 0 & \rho_e & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{at} & \varepsilon_{et} & \varepsilon_{zt} & \varepsilon_{rt} \end{bmatrix}'$$

Using the complex Schur decomposition on A and B, as in Klein (2000), we write:

$$QAZ = S$$

and

$$QBZ = T$$

where Q and Z are unitary matrices and S and T are upper triangular matrices

$$QQ' = I = ZZ'$$

The generalized eigenvalues of A and B can be found as the ratios of the diagonal elements of T and S:

$$\lambda_{ii}(B, A) = t_{ii}/s_{ii} \mid i = 1, 2 \ldots, 10$$

Let $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_m$, m be the number of eigenvalues, n_1 be the number of eigenvalues larger than one (outside unit circle), n_2 and be the number of eigenvalues smaller than one (inside unit circle). Then:

1. If $n_1 = n_2$, then there exists a unique non-explosive solution
2. If $n_1 \leq n_2$, then there exist an infinity of non-explosive solutions (multiple or sunspot solutions)

3. If $n_1 \geq n_2$, then there exist only explosive solutions (no solution)

Therefore, uniqueness requires that there are as many unstable eigenvalues as the number of control (jump) variables. Since there are four state variables in the vector s_t^0, if 4 of the generalized eigenvalues lie inside the unit circle and 6 of the generalized eigenvalues lie outside the unit circle, then the system has a unique solution.

The matrices Q, Z, S, and T are arranged so that the absolute values of the eigenvalues increase for lower rows of the matrices:

$$U = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

where Q_1 is (4×10) and Q_2 is (6×10)

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$S = \begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix}$$

$$T = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix}$$

where Z_{11}, S_{11}, and T_{11} are (4×4), Z_{12}, S_{12}, and T_{12} are (4×6), Z_{21} is (6×4), and Z_{22}, S_{22}, and T_{22} are (6×6). We define $s_t^1 = Z' s_t^0$.

$$s_t^1 = \begin{bmatrix} s_{1t}^1 \\ s_{2t}^1 \end{bmatrix}$$

and

$$s_{1t}^1 = Z_{11}' \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \\ \hat{q}_{t-1} \end{bmatrix} + Z_{21}' \begin{bmatrix} \hat{x}_t \\ \hat{g}_t \\ \hat{\lambda}_t \\ \hat{y}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix}$$

$$s_{2t}^1 = Z'_{12} \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \\ \hat{q}_{t-1} \end{bmatrix} + Z'_{22} \begin{bmatrix} \hat{x}_t \\ \hat{g}_t \\ \hat{\lambda}_t \\ \hat{y}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix}$$

Since $ZZ' = I$ and $Z' = Z^{-1}$; $s_t^0 = Zs_t^1$, we can now write the initial equation $AE_t s_{t+1}^0 = Bs_t^0 + C\xi_t$ as:

$$AZE_t s_{t+1}^1 = BZs_t^1 + C\xi_t$$

and if we pre-multiply this with Q:

$$SE_t s_{t+1}^1 = Ts_t^1 + QC\xi_t$$

We write the partitioned matrix as:

$$S_{11}E_t s_{1t+1}^1 + S_{12}E_t s_{2t+1}^1 = T_{11}s_{1t}^1 + T_{12}s_{2t}^1 + Q_1 C\xi_t$$

and with explosive eigenvalues:

$$S_{22}E_t s_{2t+1}^1 = T_{22}s_{2t}^1 + Q_2 C\xi_t$$

To keep the last equation from generating an explosive path, we need:

$$s_{2t}^1 = -T_{22}^{-1} R\xi_t$$

Where R is a (6×4) matrix given by:

$$vec(R) = vec\sum_{j=0}^{\infty}(S_{22}T_{22}^{-1})^j Q_2 C P^j = [I - P \otimes (S_{22}T_{22}^{-1})]^{-1} vec(Q_2 C)$$

Thus, we get finally using s_{2t}^1:

$$
\begin{bmatrix} \hat{x}_t \\ \hat{g}_t \\ \hat{\lambda}_t \\ \hat{y}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix} = -(Z'_{22})^{-1} Z'_{12} \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \\ \hat{q}_{t-1} \end{bmatrix} - (Z'_{22})^{-1} T_{22}^{-1} R \xi_t
$$

Z is unitary so that $Z'Z = I$:

$$
\begin{bmatrix} Z'_{11} & Z'_{21} \\ Z'_{12} & Z'_{22} \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}
$$

$$
Z'_{12} Z_{11} + Z'_{22} Z_{21} = 0
$$

$$
-(Z'_{22})^{-1} Z'_{12} = Z_{21} Z_{11}^{-1} = M_1
$$

such that:

$$
M_1 = Z_{21} Z_{11}^{-1}
$$

Furthermore,

$$
Z'_{12} Z_{12} + Z'_{22} Z_{22} = I
$$

$$
(Z_{22})^{-1} = Z_{22} + (Z_{22})^{-1} Z'_{12} Z_{12} = Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}
$$

Thus, M_2 is:

$$
M_2 = -[Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}] T_{22}^{-1} R
$$

Consequently:

$$
\begin{bmatrix} \hat{x}_t \\ \hat{g}_t \\ \hat{\lambda}_t \\ \hat{y}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix} = M_1 \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \\ \hat{q}_{t-1} \end{bmatrix} M_2 \xi_t
$$

We get a second equation using s^1_{1t}:

$$s^1_{1t} = (Z'_{11} - Z'_{21}Z_{21}Z_{11}^{-1}) \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \\ \hat{q}_{t-1} \end{bmatrix} - Z'_{21}[Z_{22} - Z_{21}Z_{11}^{-1}Z_{12}]T_{22}^{-1}R\xi_t$$

Using

$$Z'_{11}Z_{11} + Z'_{21}Z_{21} = I$$

or

$$Z'_{11} + Z'_{21}Z_{21}Z_{11}^{-1} = Z_{11}^{-1}$$

and

$$Z'_{21}[Z_{22} - Z_{21}Z_{11}^{-1}Z_{12}] = Z'_{21}Z_{22} - Z'_{21}Z_{21}Z_{11}^{-1}Z_{12} = -Z_{11}^{-1}Z_{12}$$

s^1_{1t} can be re-written as:

$$s^1_{1t} = Z_{11}^{-1} \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \\ \hat{q}_{t-1} \end{bmatrix} + Z_{11}^{-1}Z_{12}T_{22}^{-1}R\xi_t$$

Then;

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{r}_t \\ \hat{q}_t \end{bmatrix} = M_3 \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \\ \hat{q}_{t-1} \end{bmatrix} + M_4\xi_t$$

where

$$M_3 = Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}$$

$$M_4 = Z_{11}S_{11}^{-1}(T_{11}Z_{11}^{-1}Z_{12}T_{22}^{-1}R + Q_1C + S_{12}T_{12}^{-1}RP - T_{12}T_{22}^{-1}R) - Z_{12}T_{22}^{-1}RP$$

Finally, the model's solution is:

$$s_{t+1} = \Pi s_t + W\varepsilon_{t+1}$$

and

$$f_t = U s_t$$

where

$$s_t = \begin{bmatrix}\hat{y}_{t-1} & \hat{\pi}_{t-1} & \hat{r}_{t-1} & \hat{q}_{t-1} & \hat{a}_t & \hat{e}_t & \hat{z}_t & \varepsilon_{rt}\end{bmatrix}'$$
$$f_t = \begin{bmatrix}\hat{x}_t & \hat{g}_t & \hat{\lambda}_t & \hat{y}_t & \hat{\pi}_t & \hat{q}_t\end{bmatrix}'$$
$$\varepsilon_t = \begin{bmatrix}\varepsilon_{at} & \varepsilon_{et} & \varepsilon_{zt} & \varepsilon_{rt}\end{bmatrix}'$$
$$\Pi = \begin{bmatrix}M_3 & M_4 \\ 0 & P\end{bmatrix}$$
$$W = \begin{bmatrix}0 \\ I\end{bmatrix}$$
$$U = \begin{bmatrix}M_1 & M_2\end{bmatrix}$$

B Producing Smoothed Estimates for the Model's Parameters

Following Hamilton (1994) pp. 394-397, a sequence of smoothed estimates, named $\{\hat{s}_{t|T}\}_{t=1}^{T}$ is generated for the unobservable state vector, where we define for $t = 1 \ldots T$ and $j = 0, 1$:

$$\hat{s}_{t|T} = E(s_t | d_T, d_{T-1}, \ldots, d_1)$$

$$\hat{s}_{t|t-j} = E(s_t | d_{t-j}, d_{t-j-1}, \ldots, d_1)$$

$$\Sigma_{t|t-j} = E(s_t - \hat{s}_{t|t-j})(s_t - \hat{s}_{t|t-j})'$$

$$u_t = d_t - \hat{d}_{t|t-1} = C(s_t - \hat{s}_{t|t-1})$$

$$E u_t u_t' = C \Sigma_{t|t-1} C' = \Omega_t$$

The starting values are:

$$\hat{s}_{1|0} = E s_1 = 0_{(8\times1)}$$

$$vec(\Sigma_{1|0}) = vec(E s_1 s_1') = [I_{(64\times64)} - A \otimes A]^{-1} vec(BVB')$$

These values are used to produce the following sequences recursively:

$$u_t = d_t - C\hat{s}_{t|t-1}$$

$$\hat{s}_{t|t} = \Sigma_{t|t-1} C' (C \Sigma_{t|t-1} C')^{-1} u_t = \hat{s}_{t|t-1}$$

$$\hat{s}_{t+1|t} = A \hat{s}_{t|t}$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1} C' (C \Sigma_{t|t-1} C')^{-1} C \Sigma_{t|t-1}$$

$$\Sigma_{t+1|t} = BVB' + A \Sigma_{t|t} A'$$

According to Hamilton (1994), we construct a sequence $\{J_t\}_{t=1}^{T-1}$ using the equation:

$$J_t = \Sigma_{t|t} A' \Sigma_{t+1|t}^{-1}$$

and from the terminal condition which states that $\hat{s}_{T|T}$ is the last element of $\{\hat{s}_{t|t}\}_{t=1}^{T}$, the rest of the sequence is generated recursively using the following equation for $j = 1 \ldots T-1$:

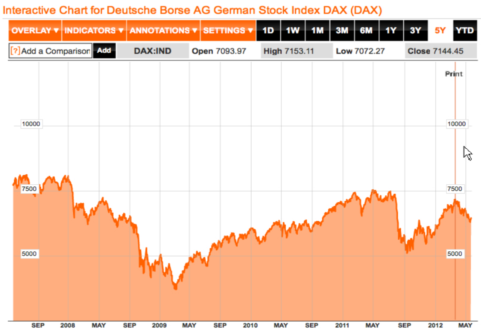
$$\hat{s}_{T-j|T} = \hat{s}_{T-j|T-j} + J_{T-j}(\hat{s}_{T-j+1|T} - \hat{s}_{T-j+1|T-j})$$

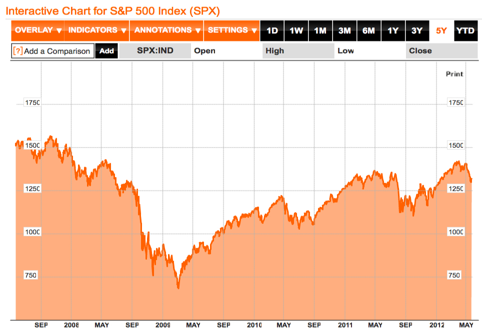
A final caveat needs to be mentioned, concerning the case in which $\Sigma_{t+1|t}$ is not invertible, as is the case in this model. In this situation, we can replace the inverse with the pseudo inverse, as shown by Kohn and Ansley (1983). The equations used to generate the sequence for $\{\Sigma_{t|T}\}_{t=1}^{T}$ are:

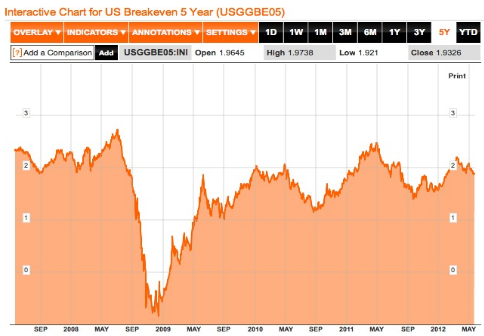
$$J_t = \Sigma_{t|t} A' \Sigma_{t+1|t}^{+}$$

$$\Sigma_{T-j|T} = \Sigma_{T-j|T-j} + J_{T-j}(\Sigma_{T-j+1|T} - \Sigma_{T-j+1|T-j}) J'_{T-j}$$

C Measures of Expected Future Inflation







These graphs show the evolution of the main stock market indexes for Germany (DAX) and US (S&P500) in the last 5 years, as well as the evolution on 5-Year breakevens, which are calculated by subtracting the real yield of the inflation linked maturity curve from the yield of the closest nominal Treasury maturity. The result of this is the implicit inflation rate for the given maturity, in this case 5 years.

What the graphs tell us is that there is no exuberance in the equity market in Europe or the US, as headline indexes have barely reached back to their pre-crisis level. Furthermore, inflation expectations for the next five years, as given by the breakevens, are hovering around 1.3% in Germany and under 2% in the US.

There is no apparent danger of an unexpected spike in inflation, as market expectations of this are clearly subdued.[5]

[5] Source: *Bloomberg*